

## **Title: ParameTRICKS**

### **Brief Overview:**

This lesson is primarily a graphing activity in which students will manipulate equations of a circle in order to translate the center to another point. They will also manipulate the T-step to obtain different polygons, and by changing the value of T in the equation they will rotate the polygon.

### **Link to Standards:**

- **Problem Solving** Students will predict changes in the equation from changes in the graph. They will match graphs and data. Students will use their tables to solve problems.
- **Algebra** Students will develop parametric equations for their data. Students will have the opportunity to make the connections between graphing in the rectangular coordinate system and parametrics.
- **Reasoning** Students will look for patterns by interpreting graphs, make conjectures about the central angles of polygons, and write a set of parametric equations for the polygon.

### **Grade/Level:**

Grades 9-12

### **Duration/Length:**

This lesson will take 1-2 periods (90 min.).

### **Prerequisite Knowledge:**

Students should be familiar with the concept of parametric equations, and using the TI-82 calculator. They should have experience with entering data, graphing, adjusting windows, adjusting mode and format, and identifying polygons.

### **Objectives:**

Students will:

- write the equation of a circle in parametric form.
- manipulate the equation of a circle with center (0,0) so that the center is new point (h,k).
- use the equation of a circle and the appropriate windows to graph and obtain regular polygons.
- match equations of circles to given graphs.
- rotate points on a graph.

### **Materials/Resources/Printed Materials:**

- TI-83 graphics calculator
- Student Worksheets (3 activities)
- Assessment sheet

**Development/Procedures:**

- Student will read the activity sheet.
- Teacher will guide students through the first problem of each activity.
- Students will solve problems working in pairs, while the teacher circulates and monitors their progress.
- Teacher will only offer additional assistance to groups that fail to arrive at an appropriate conclusion.

**Evaluation:**

At a group level, students will sketch graphs from given data and complete questions and tables on worksheets. They will be given a quiz to determine their understanding individually.

**Extension/Follow Up:**

1. Students could obtain shapes other than regular polygons by manipulating the window where the T-step is not a factor of 360.
2. As the number of sides increase, a polygon will approach a circle. By finding areas of these polygons, students can discover pi. {radius is a constant of 1}
3. Rotation of shapes through given angles.
4. Discussions of position, direction, and speed of a particle at time t.
5. Reflection of shape about the axes.

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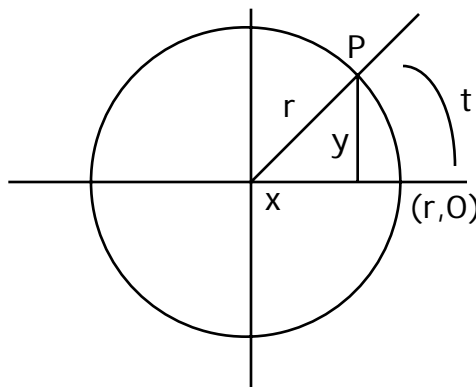
## Activity 1

### Going in Circles

The two basic circular functions, *sine* and *cosine*, are defined using a circle of radius  $r$  centered at the origin. In the figure, a length  $t$  is measured counter-clockwise around the circle from the point  $(r,0)$  to the point  $P$ , giving the angle of  $t$  radians at the origin. If  $P$  has coordinates  $(x,y)$ , then we define

$$\begin{aligned} x &= r \cos t \\ y &= r \sin t \end{aligned}$$

$x$  and  $y$  are parametric equations of the circle

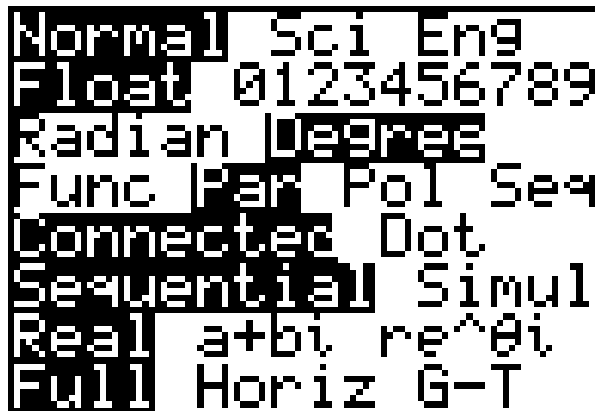


1. Give the parametric equations for a circle with radius 6 and center  $(0,0)$ .

$$X_{1T} = \underline{\hspace{2cm}}$$

$$Y_{1T} = \underline{\hspace{2cm}}$$

In order to graph these on your calculator you must have your calculator in the parametric mode.



Press  $\boxed{y=}$  and enter both equations.

```

Plot1 Plot2 Plot3
X1T=6cos(T)
Y1T=6sin(T)
X2T=
Y2T=
X3T=
Y3T=
X4T=

```

Using the following window

```

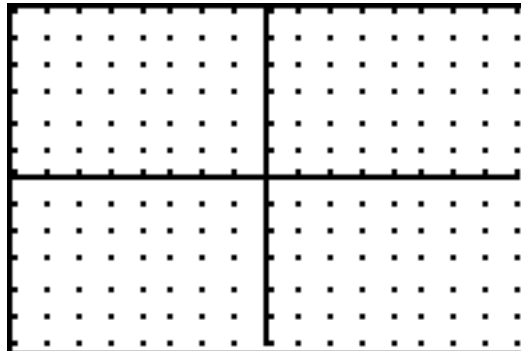
WINDOW
Tmin=0
Tmax=360
Tstep=5
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10

```

$Y_{\max} = 10$

Press **ZOOM 5**.

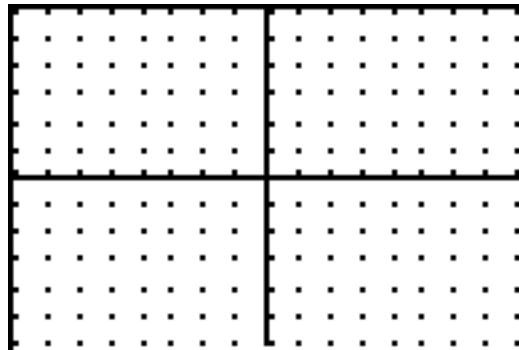
Sketch your graph:



2. Give the parametric equations and sketch the graph for a circle with center at the origin and radius 2 3 units.

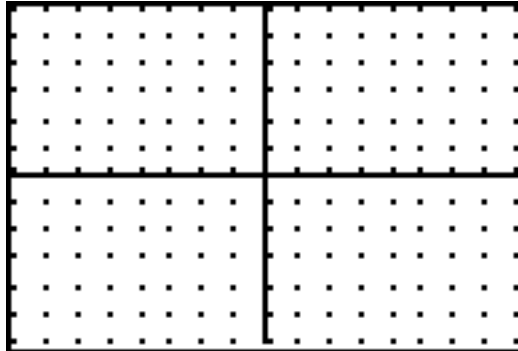
$X_{1T} = \underline{\hspace{2cm}}$

$Y_{1T} = \underline{\hspace{2cm}}$



3. Graph the following circle:  $X_{1T} = 6\cos t + 2$   
 $Y_{1T} = 6\sin t + 4$

Sketch the graph:



What is the center of the new circle? \_\_\_\_\_

CLEAR ALL EQUATIONS!

4. Graph the following circles.

Circle 1

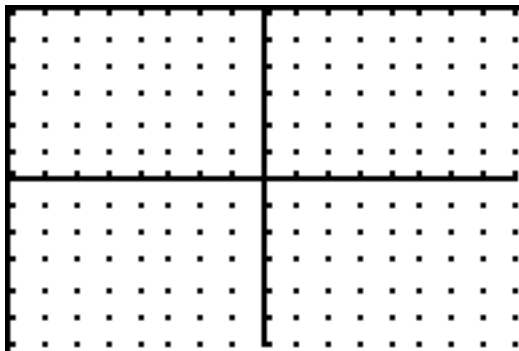
$$X_{1T} = 3\cos t$$

$$Y_{1T} = 3\sin t$$

Circle 2

$$X_{2T} = 3\cos t + 3$$

$$Y_{2T} = 3\sin t + 4$$



State the radii and centers of the circles.

Circle 1: radius \_\_\_\_\_

center \_\_\_\_\_

Circle 2: radius \_\_\_\_\_

center \_\_\_\_\_

5. The equations for the circle with center (0,0) and radius 6 are:

$$X_{1T} = 6\cos t \quad Y_{1T} = 6\sin t.$$

Write the equations for the circle with the same radius but new center at (5,4).

$$X_{2T} = \underline{\hspace{2cm}}$$

$$Y_{2T} = \underline{\hspace{2cm}}$$

6. Complete the table:

radius	center	$X_{1T} = ?$	$Y_{1T} = ?$
2	( 0,0)		
7	(-2,4)		
3	(-8,9)		
5	(8,-2)		
6	(0,-9)		
r	(a,b)		

## Activity 2

### Placing Equidistant Points on a Circle

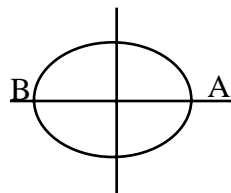
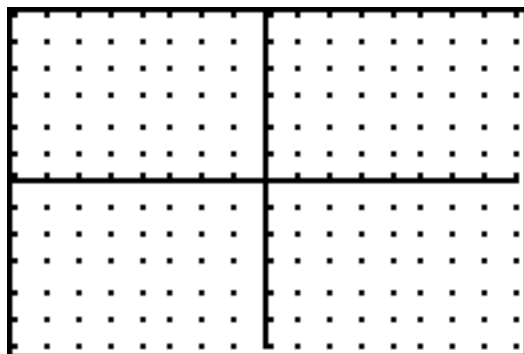
Review: Your calculator must be set for the following modes:

```

Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
    
```

- Graph the circle of radius 6 and center (0,0) with T-step = 5.  
Be sure to press **ZOOM** 6, then **Zoom** 5.

$X_{1T} = \underline{\hspace{2cm}}$        $Y_{1T} = \underline{\hspace{2cm}}$



Press **TRACE**, the coordinates of A are (\_\_, \_\_), T = \_\_\_\_

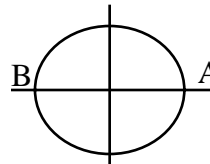
Press  and move pixel to B, coordinates of B are (\_\_, \_\_), and T = \_\_\_\_

### INSERTING TWO POINTS

Press 2nd **FORMAT**, arrow down to **AXES OFF** and press **ENTER**. Press **GRAPH**

#### SETTING T-STEP:

Between points A and B, T has changed by  $180^\circ$ .  
Press **MODE** and arrow down to **DOT**.



#### WHAT IS T STEP?

This is the change in T between two consecutive points.  
The T-step for AB is  $180^\circ$ .

Press **WINDOW**, change T-step to  $180^\circ$ .  
Press **GRAPH**.

What do you see?

Verify the points. Press TRACE the coordinates of A (\_\_\_\_,\_\_\_\_)  
Press → the coordinates of B (\_\_\_\_,\_\_\_\_)

### INSERTING 3 POINTS

Change T-Step to 120.  
Press **GRAPH** and note your observation. What do you see?

Press **MODE** and arrow down to **CONNECTED**. Press **GRAPH**. What do you see?

Working with your partner, complete the table below.

T-steps	Number of points	Name of polygon
120		
90		
72		
60		
$\frac{360}{7}$		
45		
40		
36		

## SETTING UP AND VIEWING A TABLE OF T-STEPS

Let  $Y = \text{T-step}$

Let  $X = \text{number of dots}$ .

Write an equation for  $Y$  in terms of  $X$ .

$Y = \underline{\hspace{2cm}}$

### SET UP A T-STEP TABLE

Press **MODE**, arrow down to **FUNC** and press **ENTER**.

Press **Y=**. Clear all equations.

Enter  $Y_1 = 360 / x$ .

Press 2nd **TBLSET** and set up table as shown.

TABLE SETUP		
TblStart=	1	■
ΔTbl=	1	
Indent:	Auto	Ask
Depend:	Auto	Ask

Press 2nd **TABLE**

Use your table to complete.

Number of sides	T-Step
13	<u>          </u>
<u>        </u>	16.364
28	<u>          </u>
<u>        </u>	14.4
<u>        </u>	10.286

What is the T-Step for a polygon with 59 sides?                     

**CHALLENGE!** What shape do you observe as the number of points increase?

### Activity 3

#### POLYEXERCISE - ROTATING POLYGONS

I. Inscribing an equilateral triangle in a circle with radius 6 units..

Review: The parametric equations for the circle are:

$$X_{1T} = \underline{\hspace{2cm}} \quad Y_{1T} = \underline{\hspace{2cm}}$$

$$T\text{-step} = \underline{\hspace{2cm}}$$

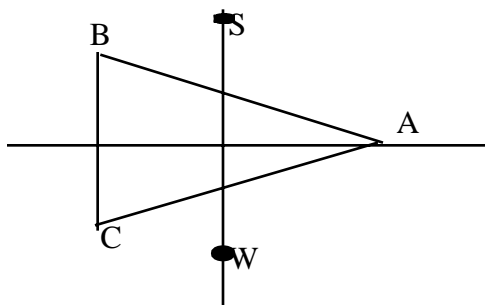
- Turn on the calculator.
- Turn axes on by pressing 2nd **FORMAT** arrow down to **AXES**, press **ENTER**.
- You will need to be in the parametric mode. Press **MODE**, arrow down to **PAR**, and press **ENTER**. Be sure at this time that **CONNECTED** and **DEGREE** are also highlighted.
- Press  $\boxed{Y=}$  and clear all equations.
- Enter the equations:  $X_{1T} = 6 \cos T$       $Y_{1T} = 6 \sin T$
- Press **ZOOM 6**.
- Adjust T-step to 120 (T-step is in **WINDOW**)
- Press **ZOOM 5**.

II. Labelling the Triangle:

Press **TRACE**     pixel is at  $T = 0$ , name this point A.

Press  $\boxed{\rightarrow}$      pixel is at  $T = 120$ , name this point B

Press  $\boxed{\rightarrow}$      pixel is at  $T = 240$ , name this point C



III. Moving B to the positive Y axis (That is to the line containing the point S)

In moving from B to S we are moving from  $T=120$  to  $T=90$ . We are going back  $30^\circ$ , that is T had been reduced by  $30^\circ$ . Our new value for T is  $(T - 30)$ .

Our new parametric equations are:

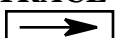
$$X_{1T} = 6 \cos (T - 30) \quad Y_{1T} = 6 \sin (T - 30)$$

DOES IT WORK?

Enter your equations and graph.

Is B in the correct position?

Press **TRACE**

Press  (T = 120 is on the positive Y-axis)

IV. Moving B to the negative X-axis (to R)

At B  $T = 120$

At R  $T = 180$

In moving from B to R, T increases by 60 degrees, therefore we will need to add 60 degrees to T.

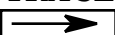
Our new parametric equations are:  $X_{1T} = 6 \cos (T + 60)$   $Y_{1T} = 6 \sin (T + 60)$

Press 

Enter your parametric equations and graph.

Is B in the correct position?

Press **TRACE**

Press  (T120 is on the negative x axis)

V. Moving vertex C to the negative Y axis (in the same line with the point W).

The value of T at C = \_\_\_\_\_

The value of T at W = \_\_\_\_\_

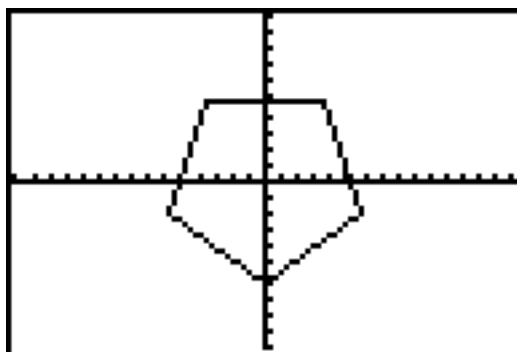
Therefore the change in T is \_\_\_\_\_

Our new parametric equations are  $X_{1T} = \underline{\hspace{2cm}}$   $Y_{1T} = \underline{\hspace{2cm}}$

If C is in the correct position, then  $T = \underline{\hspace{1cm}}$  must be on the negative y - axis.



CHALLENGE: Can you find the equations necessary to obtain the following graph?



## ASSESSMENT

Quiz

Name \_\_\_\_\_

Period \_\_\_\_\_

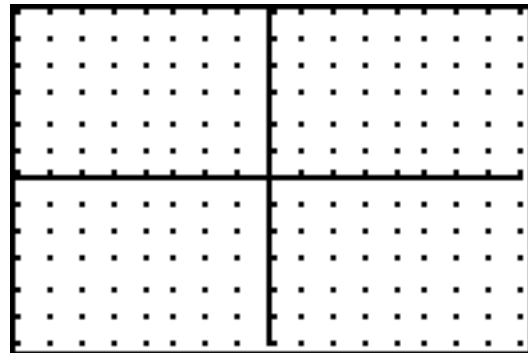
Date \_\_\_\_\_

Part A:

1. Write the parametric equations of the circle with center (0,0) and radius 3. Sketch the graph.

$$X_{1T} = \underline{\hspace{2cm}}$$

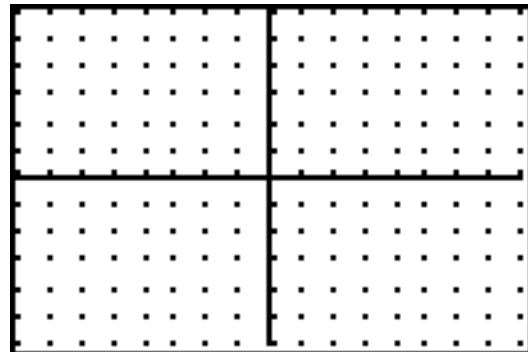
$$Y_{1T} = \underline{\hspace{2cm}}$$



2. Give the parametric equation of a circle with center (3,4) and radius 5 units. Sketch the graph.

$$X_{1T} = \underline{\hspace{2cm}}$$

$$Y_{1T} = \underline{\hspace{2cm}}$$



3. The parametric equations of a circle of radius  $r$  and center (0,0) are:

$$X_{1T} = \underline{\hspace{2cm}}$$

$$Y_{1T} = \underline{\hspace{2cm}}$$

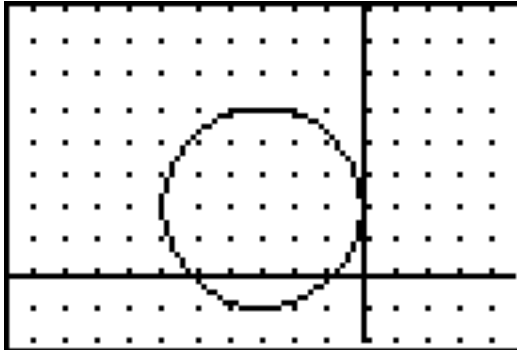
If we need to translate the center to the point (h,k). Then our parametric equations become:

$$X_{1T} = \underline{\hspace{2cm}} \quad Y_{1T} = \underline{\hspace{2cm}}$$

Part B:

Describe each circle and write the corresponding parametric equations.

4.

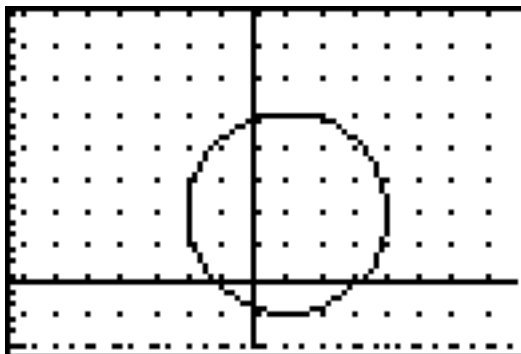


radius = \_\_\_\_\_

center = \_\_\_\_\_

$X_{1T} =$  \_\_\_\_\_  $Y_{1T} =$  \_\_\_\_\_

5.

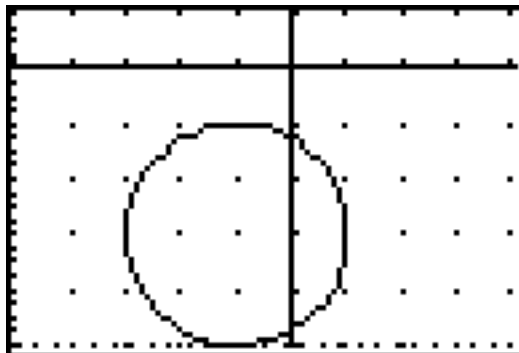


radius = \_\_\_\_\_

center = \_\_\_\_\_

$X_{1T} =$  \_\_\_\_\_  $Y_{1T} =$  \_\_\_\_\_

6.



radius = \_\_\_\_\_

center = \_\_\_\_\_

$X_{1T} =$  \_\_\_\_\_  $Y_{1T} =$  \_\_\_\_\_

7. Write the equations of a circle of radius 8 and center (0,0).

$$X_{1T} = \underline{\hspace{2cm}} \quad Y_{1T} = \underline{\hspace{2cm}}$$

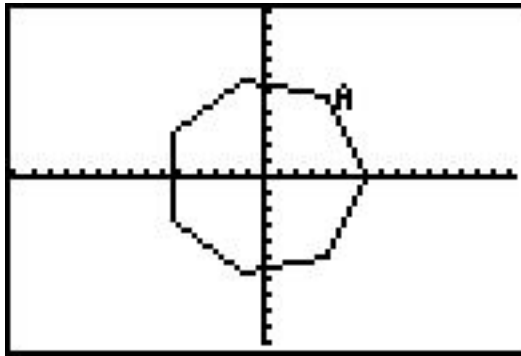
Inscribe a regular hexagon in this circle. T-step =                 

8. Inscribe a square in problem 1. T-step =                 .

9. A T-step of 45 will inscribe a regular hexagon in a given circle. True/ False.

10. A T-step of          will inscribe a 14 sided regular polygon in a given circle.

11. Write the parametric equations for the graph shown below.



$$X_{1T} = \underline{\hspace{2cm}} \quad Y_{1T} = \underline{\hspace{2cm}}$$

T-step =                 

Move vertex A to the positive y axis. The new parametric equations are:

$$X_{1T} = \underline{\hspace{2cm}} \quad Y_{1T} = \underline{\hspace{2cm}}$$

## NOTES TO TEACHER


### ACTIVITY I:


- A. Center of Circle:      x coordinate of center is at  $T = 90$   
   y coordinate of center is at  $T = 0$ .
2. Pixel will only move between 0 and 360. If your arrow passes a given point, you will have to arrow back to that point.

### ACTIVITY II:

To obtain points you have to be in **dot** mode. To obtain the polygons you have to be in **connected** mode.

### ACTIVITY III:

To highlight T on the TI - 82 you have to press **TRACE**  .

In all activities where T is needed on the TI - 82, replace Press **TRACE** with  
press **TRACE**  .

To turn on the axes on the TI-82, press WINDOW and arrow over to FORMAT.

For graphs 3 (Activity I) and VI (Activity III): in sketching graphs, it will be necessary for each tic mark to represent 2 units.